	2nd Exam	
Name:	Department:	GRADE
Student No:	Course: Calculus I	
Signature:	Date: 20/12/2017	

Each problem is worth 20 points. Demonstrate your solution steps clearly.

1. $\int_0^{\infty} \frac{dx}{(1+x^2)(1+\arctan x)} =$ $\ln(1+\pi/2)$

$1+\arctan x = u \Rightarrow \frac{dx}{1+x^2} = du$

$\int \frac{dx}{(1+x^2)(1+\arctan x)} = \int \frac{du}{u} = \ln|u| + C = \ln|1+\arctan x| + C.$

$\lim_{R\to\infty} \int_0^R \frac{dx}{(1+x^2)(1+\arctan x)} = \lim_{R\to\infty} \ln|1+\arctan R| - \ln|1+\arctan 0| = \ln|1+\pi/2| - \ln 1.$ ($\arctan 0 \to 0$)

2. Sketch the graph of the function $f(x) = 4x^3 - x^4$ by first making a table of the intervals on which f increases/decreases and concave up/down.

$f'(x) = 12x^2 - 4x^3 = 4x^2(3-x) = 0 \Rightarrow x = 0,\ x = 3$

$f''(x) = 24x - 12x^2 = 12x(2-x) = 0 \Rightarrow x = 0,\ x = 2.$

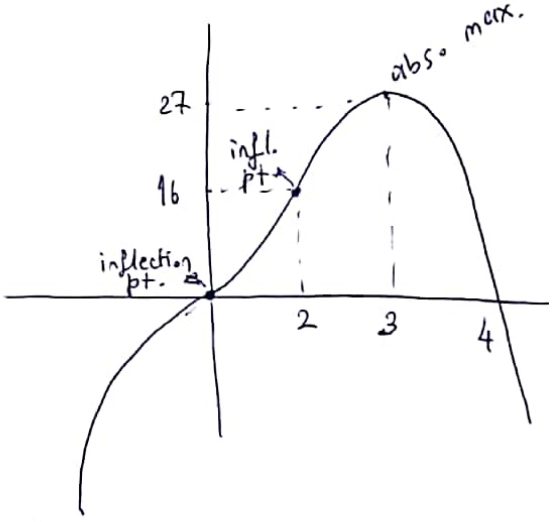
x		0		2		3	
$f'(x)$	+		+		+		−
$f''(x)$	−		+		−		−
$f(x)$	⌒		⌣		⌒		⌒

$f(0) = 0$

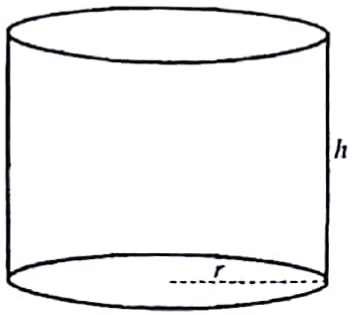
$f(2) = 16$

$f(3) = 27$

$f(4) = 0$



3. Find the radius r and height h of the cylindirical tin with volume 2π and minimum surface area.



$r=1 \qquad h=2$

$V = \pi r^2 h = 2\pi \qquad r^2 h = 2$

$A = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \cdot \frac{2}{r^2}$

$A(r) = 2\pi\left(r^2 + \frac{2}{r}\right)$

$A'(r) = 2\pi\left(2r - \frac{2}{r^2}\right) = 0 \Leftrightarrow 2r = \frac{2}{r^2} \Rightarrow r^3 = 1 \Rightarrow r = 1. \Rightarrow h = 2.$

r		0		1
$A'(r)$	$\|$	$\|\|$	$-$	0 $+$

$A(1) = 6\pi$ is minimum.

4. $\int \sin(\ln x)\,dx =$ $\frac{x}{2}\left(\sin(\ln x) - \cos(\ln x)\right) + C$

$I = \int \underbrace{\sin(\ln x)}_{u} \underbrace{dx}_{dv} = \sin(\ln x)\,x - \int \underbrace{\cos(\ln x)}_{u} \cdot \frac{1}{x} \cdot x \underbrace{dx}_{dv}$

$= \sin(\ln x)\,x - \left[\cos(\ln x)\,x - \int -\sin(\ln x) \cdot \frac{1}{x}\, x\,dx\right]$

$I = x\left(\sin(\ln x) - \cos(\ln x)\right) - I.$

5. $\int \frac{x^2}{(x-1)(x^2+2x+1)}\,dx =$ $\frac{1}{4}\ln|x-1| + \frac{3}{4}\ln|x+1| + \frac{1}{2}\frac{1}{(x+1)} + C$

$\frac{x^2}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$x^2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$

$x = 1 \Rightarrow \quad 1 = 4A \quad \Rightarrow A = 1/4$

$x = -1 \Rightarrow \quad 1 = -2C \quad \Rightarrow C = -1/2$

$x = 0 \Rightarrow \quad 0 = A - B - C = \frac{1}{4} - B + \frac{1}{2} \Rightarrow B = 3/4$

$\text{integral} = \int \frac{1/4\,dx}{x-1} + \int \frac{3/4\,dx}{x+1} + \int \frac{-1/2\,dx}{(x+1)^2}$